

Maximum Entropy Thresholding

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Single Threshold

Let $h(i)$ be value of a normalized histogram. Typically i takes integer values from 0 to 255. We assume that $h(i)$ is normalized, that is:

$$\sum_{i=0}^{i_{max}} h(i) = 1 \quad (1)$$

Entropy of white pixels:

$$H_B(t) = - \sum_{i=0}^t \frac{h(i)}{\sum_{j=0}^t h(j)} \log \frac{h(i)}{\sum_{j=0}^t h(j)} \quad (2)$$

Entropy of black pixels:

$$H_W(t) = - \sum_{i=t+1}^{i_{max}} \frac{h(i)}{\sum_{j=t+1}^{i_{max}} h(j)} \log \frac{h(i)}{\sum_{j=t+1}^{i_{max}} h(j)} \quad (3)$$

Optimal threshold can be selected by maximizing the entropy of black and white pixels:

$$T = \underset{t=0 \dots i_{max}}{\text{Arg Max}} H_B(t) + H_W(t) \quad (4)$$

Multiple Threshold

Assume that we want to find optimal n thresholds, the Equation (4) can be generalized from one threshold to n threshold as follows:

$$\{T_1, \dots, T_n\} = \underset{t_1 < \dots < t_n}{\text{Arg Max}} H(-1, t_1) + H(t_1, t_2) + \dots + H(t_n, i_{max}) \quad (5)$$

Where

$$H(t_k, t_{k+1}) = - \sum_{i=t_k+1}^{t_{k+1}} \frac{h(i)}{\sum_{j=t_k+1}^{t_{k+1}} h(j)} \log \frac{h(i)}{\sum_{j=t_k+1}^{t_{k+1}} h(j)} \quad (6)$$

References

1. J.N. Kapur, P.K. Sahoo and A.K.C. Wong, "A New Method for Gray-Level Picture Thresholding Using the Entropy of the Histogram", *CVGIP*, (29), pp.273-285, 1985.